

1. Abstract

Nonmigrating tidal diagnostics of SABER temperature observations in the Mesosphere and lower Thermosphere (MLT) region reveal significant variability on time-scales of a few days to weeks. Therefore, it is very important to assess short-term variability and see its role in the variability of various state variables in the MLT. This work presents a statistical diagnostics of short term tidal variability using a novel approach leveraging Information Theory and Bayesian statistics. The overarching goal is to develop an empirical model, that is, to diagnose and predict short-term tidal variability as a function of atmospheric drivers such as QBO, ENSO, solar cycle, etc.; using time dependent probability density functions, Shannon entropy and Kullback-Leibler divergence. In this paper, we show the statistical approach to establish the framework and exemplify initial results with emphasis on how short-term tidal variability changes on annual and interannual timescales, including its response to the Quasi-Biennial Oscillation (QBO).

Objective:
Understand the causes of short-term tidal variability and its response to atmospheric variability such as the QBO.

2. Methodology

What is the short-term tidal variability as a function of year, season, latitude and altitude ?

- SABER tidal data (X timeseries)
1. SABER tidal data (daily) (tidal deconvolution method)
 2. Data gap handling (Lomb-Scargle method) [3]
 3. Removal of long-term variability to keep variability on shorter scale (less than 31 days)

Time dependent probability density function (TDPDF)

Choose Sampling window e.g., W=365, 181, 61, 31 days

Move window by time step of one day until the end of time series

Calculate probability density function (analogous of histogram)

Optimal binning scheme by Bayesian theorem

1. Empirical orthogonal functional analysis of TDPDF
2. Information-theoretic techniques on TDPDF

Structural analysis of TDPDF

Shannon Entropy (SE) • Measure of Information content in TDPDF

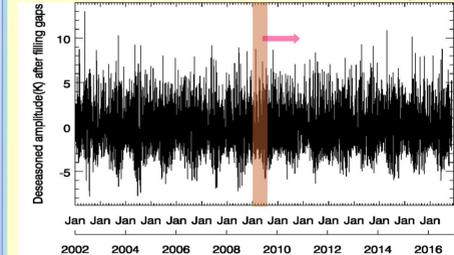
Kullback-Leibler divergence (KLD) • Relative entropy or divergence measure between two PDFs in TDPDF • A tool for future and past prediction

How is short-term tidal variability studied using a sample of such large window length (e.g. 181 days)?

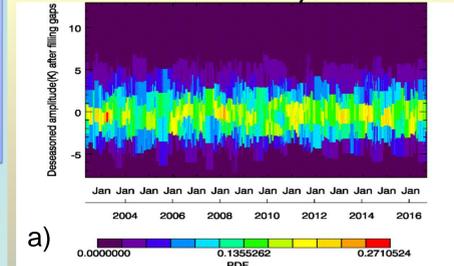
The key point is that we consider here the 'variability over variability' in TDPDF. The TDPDF using 181-day window is utilized to study inter-annual changes of the short-term tidal variability. These changes represent how short-term variability responds to other forcing factors in the atmosphere on inter-annual or larger scales. Identifying changes in short-term tidal variability related to intra-seasonal effects (such as MJO, a ~60-day periodic atmospheric event) would require a shorter window length sample selection.

3. KLD: Annual Variations

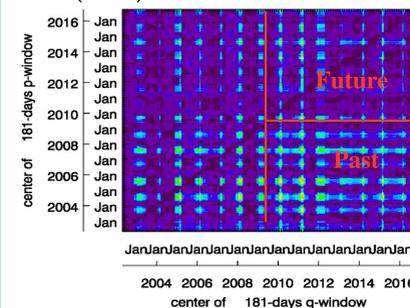
SABER DE3 Data @95 km and equator



TDPDF for 181-day window



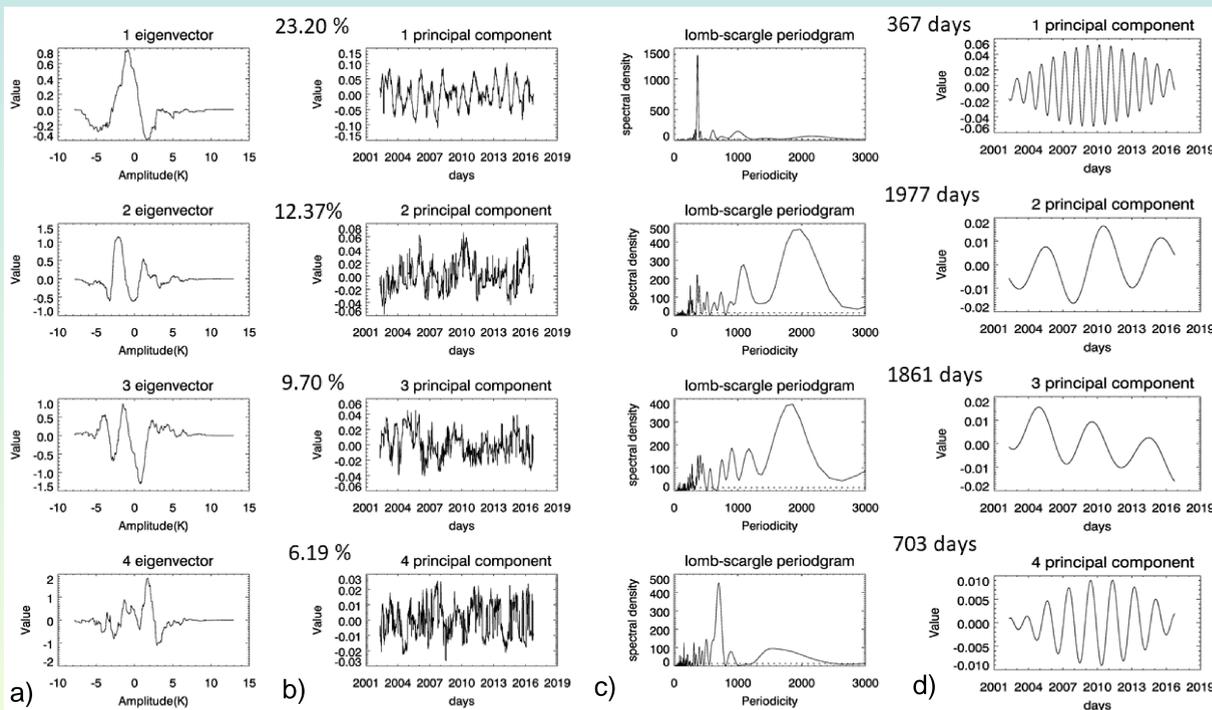
Kullback-Leibler Divergence (KLD)



b)

$$D(p(x)||q(x)) = \sum_x p(x) \log \frac{p(x)}{q(x)}$$

4. EOF Analysis of TDPDF: Interannual Variations

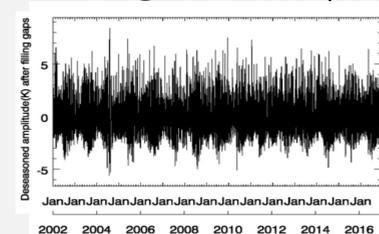


Overview of the results for SABER DE3 data at 95 km, equator: a) The TDPDF of the de-seasoned data is obtained using a sample of 181-day window length and then advancing the window in steps of one day until the end of the time series [1]. Bayesian optimal binning scheme provides the best optimization of information vs noise in the data [2]. b) The annual variation in the KLD indicates the seasonal variations of PDF structures of the short-term tidal variability. Moreover, for a given time window, points vertically above (below) the diagonal signify the ability of this windowed PDF to predict future (past) PDFs. A low KLD value indicates relative stability and high predictability.

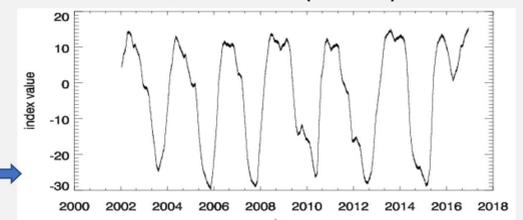
5. Ongoing Work

Under what conditions will the short-term tidal variability be largest as a function of QBO, ENSO, solar activity, stochastic weather processes? – 2D optimal binning and mutual information.

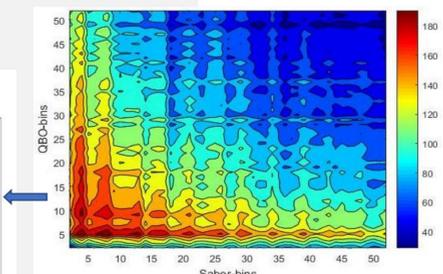
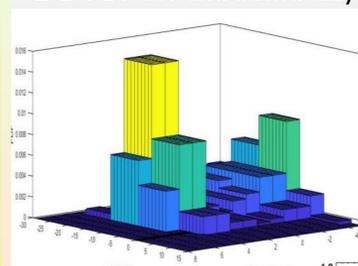
SABER Data @90 km and equator



QBO- index (NOAA)

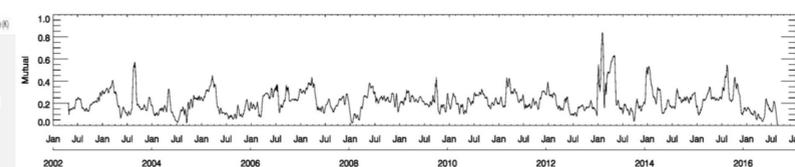


2-D PDF for the same day



Two-dimensional posterior probability for a day (2-D Optimal binning scheme by Bayesian theorem)

Mutual Information



6. Conclusion

- ✓ Preliminary results show seasonal and interannual variations in short-term tidal variability in the MLT, including a QBO signal. The latter is possibly due to reduced (enhanced) tidal dissipation in the mesosphere when the stratospheric QBO is westerly (easterly). This is because the mesospheric QBO is out of phase with the stratospheric one and eastward propagating tides are harder to dissipate in easterly mean winds due to Doppler shift towards higher frequencies [6].
- ✓ The KLD plot shows that the framework will potentially contribute in the setup of a forecast model, particularly once the ongoing work on 2D optimum binning and mutual information is completed.

7. References

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Concept of computing the mutual information ($I(X; Y, t) = D(p(x, y, t)||p(x, t)p(y, t))$) between the de-seasoned SABER DE3 (symmetric part w.r.t. equator) and the QBO index. Mutual Information (MI) is an information theoretic measure of find dependencies in two time series [4]. There are various structural changes in the MI values with time, which indicate changes in the dependency level of variability of the short-term tidal variability with the QBO index. We still need to understand the reason behind every peak and dip in the MI values: the next step is to improve the 2D optimal binning and to compute the statistical significance of the MI values. It may also be beneficial to compute the MI between relative (w.r.t. monthly mean amplitudes) short-term variability and the QBO. Afterwards, this method will potentially help to identify the true information content shared between the two timeseries.

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